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SYSTEM IDENTIFICATION FOR EVALUATING SOIL–STRUCTURE INTERACTION EFFECTS IN BUILDINGS FROM STRONG MOTION RECORDINGS

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SUMMARY

Parametric system identification is used to evaluate seismic soil–structure interaction effects in buildings. The input–output strong motion data pairs needed for evaluations of flexible- and fixed-base fundamental mode parameters are derived. Recordings of lateral free-field, foundation, and roof motions, as well as foundation rocking, are found to be necessary for direct evaluations of modal parameters for both cases of base fixity. For the common situation of missing free-field or base rocking motions, procedures are developed for estimating the modal parameters that cannot be directly evaluated. The accuracy of these estimation procedures for fundamental mode vibration period and damping is verified for eleven sites with complete instrumentation of the structure, foundation, and free-field. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: system identification; seismic soil–structure interaction

1. INTRODUCTION

The objective of system identification is to evaluate the unknown properties of a system using a known input into, and output from, that system. For applications to buildings, system identification can be used to estimate modal frequencies and damping ratios which characterize the structural response to earthquake ground motion. To provide a simple quantification of the effects of Soil–Structure Interaction (SSI) on building response, modal vibration parameters are sought which describe the behaviour of the structure alone (i.e. fixed-base parameters) and the soil–foundation–structure system (i.e. flexible-base parameters) using input–output pairs consisting of various combinations of free-field, foundation, and roof-level recordings (Figure 1). While system identification analyses are also capable of estimating mode shapes and participation factors from the strong motion data, the focus of this paper is on fundamental-mode vibration period and damping ratio for different cases of base fixity which enable simple quantifications of SSI effects on structural response, as appropriate for earthquake resistant design building code specifications.[1]

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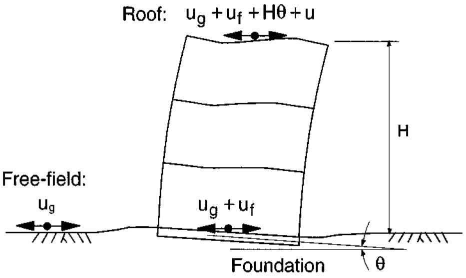

Figure 1. Motions used as inputs and outputs for system identification of structures

There are two principal system identification techniques:

1. Non-parametric procedures evaluate complex-valued transmissibility functions without fitting an underlying model. These functions are computed from smoothed power and cross-power spectral density functions of the input and output motions, and represent an estimate of output-to-input motion in the frequency domain. Modal frequencies and damping ratios are estimated from peaks in the transmissibility function amplitude.[2–4]
2. Parametric procedures develop numerical models of Laplace domain transfer functions. The parameters for these functions are estimated by minimizing the error between the model and recorded output in the discrete time domain using least-squares techniques. Modal parameters are estimated from peaks, or poles, in the transfer function amplitude surface.[5–7]

The determination of vibration frequencies and damping ratios from non-parametric procedures can be problematic (especially for damping), as the shape of transmissibility functions is dependent on the method of estimating the spectral density functions (i.e. the number of points in the fast Fourier transform and the windowing procedures).[2] Parametric procedures provide a more rigorous modelling of system response, because the transfer function for a given set of time histories is only dependent on two user-defined parameters: the delay between the input and output, and the number of modes used in the analyses (i.e. the order of the model). When these parameters are selected judiciously,[5,6] the modal frequencies and damping ratios can be reliably evaluated for linear structures.

The physical interpretation of system identification results depends on the input and output motions used. In the fixed-base case, the 'system' is the structure alone, and the differences between the input and output motions must reflect only the structure's flexibility. The flexible-base case represents the structure–foundation–soil system, and the differences between the input and output motions must reflect the flexibility of all three components.

Luco[8] determined the input–output strong motion recordings necessary to evaluate fixed- and flexible-base parameters using non-parametric system identification techniques. This was accomplished by solving the equations of motion for a multi-degree-of-freedom stucture resting on a flexible medium (represented by complex-valued lateral and rocking springs) in the frequency domain, and then using this solution to derive expressions for transmissibility functions for different input-output pairs. Safak[9] presented a procedure for identifying SSI effects in buildings for cases when no free-field motions are available and the effects of base rocking are negligible. Like Luco,[8] Safak's procedure made use of non-parametric procedures for identifying modal vibration parameters. This paper extends these earlier works to consider transfer functions expressed in the Laplace domain so that parametric system identification procedures can be utilized to identify the effects of soil–structure interaction (including both base translation and rocking).

The scope of this paper is twofold: (1) derive the input and output pairs that are needed to evaluate fixed- and flexible-base modal parameters by parametric system identification procedures, and (2) describe how

selected fundamental mode parameters can be estimated when only limited strong motion recordings are available from a site. The parameter estimation procedures are verified using system identification results from 11 sites for which an identification of both fixed- and flexible-base parameters was possible. These identification and parameter estimation procedures have been used to evaluate SSI effects from strong motion recordings in 58 buildings,[7] comprising the largest empirical data set for SSI compiled to date.

2. BASE FIXITY CONDITIONS FOR DIFFERENT INPUT–OUTPUT PAIRS

2.1. Solution of equation of motion in Laplace domain

The properties of a linear structure with n degrees of freedom include its mass matrix $\mathbf{m}$, stiffness matrix $\mathbf{k}$, and damping matrix $\mathbf{c}$. The damping matrix is intended to model energy dissipation in the structure, and is assumed to be of classical form (i.e. the damping does not couple modal responses). While modal damping can be coupled in actual structures, the classical damping assumed here is appropriate for use with simplified, code-oriented SSI analysis procedures.

The dynamic displacements of a structure relative to its base are described by the $n\times 1$ vector $\mathbf{u}$, with corresponding velocity and acceleration vectors $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$, respectively. The total displacement vector $\mathbf{u}^{t}$ is the sum of the ground displacement u_g and the dynamic displacement of the structure,

$$\mathbf{u}^{t}(t) = \mathbf{u}(t) + \mathbf{1}u_{g}(t) \tag{1}$$

where $\mathbf{1} = n\times 1$ vector of 1's. The equation of motion for the structure is[10]

$$\mathbf{m}\ddot{\mathbf{u}}(t) + \mathbf{c}\dot{\mathbf{u}}(t) + \mathbf{k}\mathbf{u}(t) = -\,\mathbf{m}\mathbf{1}\ddot{u}_{g}(t) \tag{2}$$

Solution of equation (2) in the frequency domain is well established.[10] The solution procedure in the Laplace domain is similar,[7] and hence is not repeated in detail. The solution of the parametric system identification problem is formulated to relate the total acceleration vector for various points in the structure ($\ddot{\mathbf{u}}^{t}$) to the input, or ground acceleration,

$$\ddot{\mathbf{u}}^{t}(s) = \mathbf{H}(s)\ddot{u}_{g}(s) \tag{3a}$$

where s is the complex-valued Laplace domain variable. Element j of the vector quantity $\mathbf{H}(s)$ in equation 3(a) represents the transfer function between the input and the jth output. $\mathbf{H}(s)$ is the sum of contributions from J modes (where $J < n$),

$$\begin{aligned} \mathbf{H}(s) &= \sum_{j=1}^{J} \frac{L_j^*}{m_j^*}\,\mathbf{\Phi}_j H_j(s) \\ H_j(s) &= \frac{2\zeta_j\omega_j s + \omega_j^2}{s^2 + 2\zeta_j\omega_j s + \omega_j^2} \end{aligned} \tag{3b}$$

where $\mathbf{\Phi}_j$ is the mode shape for mode j, L_j^* is the generalized influence factor, m_j^* is the generalized mass, and ω_j and ζ_j are the modal frequency and damping ratio for mode j. Different recording locations within a structure exhibit the same poles, so generally it is adequate to consider only the output at the roof for identifying parameters for the lower, most significant modes. Hence, single-output system identification analyses were used in this study, which reduces the vector $\mathbf{H}(s)$ to a scalar function for the roof response. It should be noted that contributions from all J modes are represented in the single-output solution, although only fundamental-mode parameters are significantly affected by SSI.[11]

The amplitude of a particular component of $\mathbf{H}(s)$ is a continuous surface with peaks (also known as poles) for each mode which occur at a position on the horizontal plane which can be related to modal frequencies and damping ratios. When a component of $\mathbf{H}(s)$ is evaluated along the imaginary axis, the transmissibility

function $H(\mathrm{i}\omega)$ is obtained, which gives the ratio of output-to-input acceleration as a function of frequency ω. The roof component of $\mathbf{H}(s)$ for an example structure is presented in a later section of this paper (see Figure 7).

The function $\mathbf{H}(s)$ can be estimated from strong motion accelerographs recorded at suitable input and output points using parameteric system identification techniques. For a given input–output pair, this process involves the following steps:[5,6] (1) convert the continuous time structural model represented by $\mathbf{H}(s)$ to an equivalent discrete time model, $\mathbf{H}(z)$ (where z is the Z-transform operator), (2) estimate the parameters describing $\mathbf{H}(z)$ using least-squares techniques to minimize the error between the model output and the recorded output, and (3) convert the poles of $\mathbf{H}(z)$, $z_1 \ldots z_{2J}$ back to poles $s_1 \ldots s_{2J}$ for the continuous time model. The poles of $\mathbf{H}(s)$ occur at the roots of the denominator in equation (3b). Hence, the poles are located as follows:

$$s_j, s_j^* = -\zeta_j\omega_j \pm \mathrm{i}\omega_j\sqrt{1-\zeta_j^2} \tag{4}$$

from which the modal frequencies and damping ratios can be computed as

$$\begin{aligned} \omega_j &= \sqrt{s_j s_j^*} \\ \zeta_j &= -\frac{\mathrm{Re}(s_j)}{\omega_j} \end{aligned} \tag{5}$$

2.2. Interpretation of SSI effects from modal parameters

The interpretation of the modal parameters identified by the parametric identification procedures for different input–output pairs is made with respect to the simple inertial SSI model shown in Figure 2. Single-degree-of freedom structural models are commonly employed in SSI analyses because inertial interaction effects are most pronounced in the first mode. This simple system can be viewed as a direct model of a single storey building, or more generally, as an approximate model of a multi-mode, multi-storey structure which is dominated by first-mode response. In the case of a multi-storey structure, the height h is the distance from the base to the centroid of the inertial forces associated with the first vibration mode, and displacement u is that of the centroid. The effective displacement in Figure 2 is different than the roof displacement used for single-output system identification. This use of different displacements in the single-degree-of-freedom model and the system identification does not affect the location of the poles, and fundamental mode parameters derived from the system identification procedures in Section 2.1 can be used in conjunction with the simple model in Figure 2. This model is similar to that considered by Veletsos and Nair[12] for surface foundations and Bielak[13] for embedded foundations.

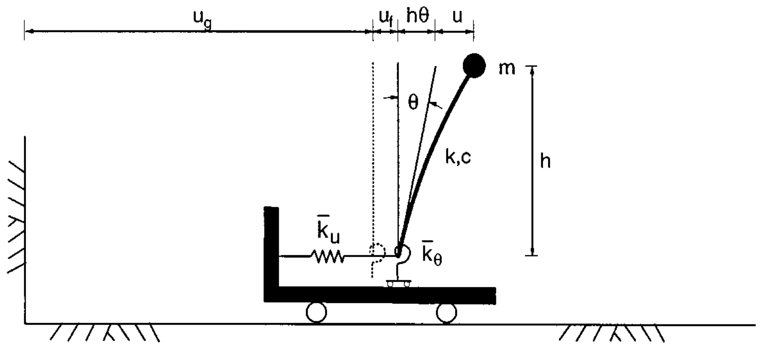

Figure 2. Simplified model for analysis of inertial interaction

The transfer functions in the Laplace domain, and the poles of these surfaces, can be derived for different input–output pairs (representing different base fixity conditions) in terms of soil, foundation, and structure properties. The equations of motion describing the simple system in Figure 2 are[14]

$$\text{lateral at } m\text{: } m(\ddot{u}_{\mathrm{f}} + h\ddot{\theta} + \ddot{u}) + c\dot{u} + ku = -m\ddot{u}_{\mathrm{g}} \tag{6a}$$

$$\text{total lateral: } m(\ddot{u}_{\mathrm{f}} + h\ddot{\theta} + \ddot{u}) + m_{\mathrm{f}}\ddot{u}_{\mathrm{f}} + c_u\dot{u}_{\mathrm{f}} + k_u u_{\mathrm{f}} = -(m + m_{\mathrm{f}})\ddot{u}_{\mathrm{g}} \tag{6b}$$

$$\text{total rotation: } mh(\ddot{u}_{\mathrm{f}} + h\ddot{\theta} + \ddot{u}) + I\ddot{\theta} + c_\theta\dot{\theta} + k_\theta\theta = -mh\ddot{u}_{\mathrm{g}} \tag{6c}$$

where I is the rotational moment of inertia of the structure, and k_u, c_u, k_θ, and c_θ are coefficients of the foundation impedance function expressing the stiffness and damping characteristics of soil/foundation interaction. Referring to Figure 2, the impedance function is defined as

$$\bar{k}_k = k_k(a_0,\upsilon) + \mathrm{i}\omega c_k(a_0,\upsilon) \tag{7}$$

where subscript k denotes either deformation mode u or θ, ω is angular frequency (rad/sec), a_0 is a dimensionless frequency defined by $a_0 = \omega r/V_{\mathrm{S}}$, r is the foundation radius, V_{S} is the soil shear wave velocity, and υ is the soil Poisson ratio.

The time-dependent motions in equation (6) are transformed to the Laplace domain according to $f(t) = \hat{f}e^{st}$. As noted previously, the coefficients k_u, c_u, k_θ, and c_θ are real-valued functions of frequency. In the Laplace domain, they are interpreted as real-values evaluated at the pole of the transfer function being sought. For practical purposes, these Laplace domain coefficients are the same as the foundation impedance in the frequency domain evaluated at the frequency of the structure for the appropriate level of hysteretic soil damping. Transforming to the Laplace domain and dividing through by the mass coefficients, equation (6) can be re-written as

$$s^2\hat{u}_{\mathrm{f}} + s^2h\hat{\theta} + A\hat{u} = -s^2\hat{u}_{\mathrm{g}} \tag{8a}$$

$$A_u\hat{u}_{\mathrm{f}} + s^2\mu h\hat{\theta} + s^2\mu\hat{u} = -s^2\hat{u}_{\mathrm{g}} \tag{8b}$$

$$s^2\hat{u}_{\mathrm{f}} + A_\theta h\hat{\theta} + s^2\hat{u} = -s^2\hat{u}_{\mathrm{g}} \tag{8c}$$

where $\mu = m/(m + m_{\mathrm{f}})$. Neglecting the rotational inertia of the structure and the mass of the foundation (i.e. $I = 0$, $\mu = 1$), the A coefficients are defined as

$$A_k = s^2 + 2\zeta_k\omega_k s + \omega_k^2 \tag{9}$$

where $k = [\ ]$, u, or θ (referring to the fixed-base structure, foundation translation, and foundation rocking, respectively), and the frequencies and damping ratios (ω_k, ζ_k) describe the dynamic behaviour of the structure (ω, ζ) or soil–foundation system (ω_u, ζ_u and ω_θ, ζ_θ). These parameters are related to the system properties as follows:

$$\begin{gathered} \omega_k^2 = \frac{k_k}{m} \quad \zeta_k = \frac{c_k}{2m\omega_k} \quad (k = [\ ] \text{ or } u) \\ \omega_\theta^2 = \frac{k_\theta}{mh^2} \quad \zeta_\theta = \frac{c_\theta}{2mh^2\omega_\theta} \end{gathered} \tag{10}$$

Equation (8) has three unknown response functions ($\hat{u}$, $\hat{u}_f$, and $\hat{\theta}$) and three equations. Hence, the response can be solved for directly in terms of the system properties,

$$\frac{\hat{u}}{\hat{u}_{\mathrm{g}}} = -\frac{B_\theta B_u s^2}{C_{\mathrm{s}}}, \quad \frac{\hat{u}_{\mathrm{f}}}{\hat{u}_{\mathrm{g}}} = -\frac{B_\theta B s^2}{C_{\mathrm{s}}} \quad \text{and} \quad \frac{h\hat{\theta}}{\hat{u}_{\mathrm{g}}} = -\frac{BB_u s^2}{C_{\mathrm{s}}} \tag{11}$$

where $B_k = A_k - s^2$ and $C_s = s^2(B_uB + B_uB_\theta + B_\theta B) + B_uBB_\theta$.

Equation (11) represents the complete solution for the SSI model in Figure 2, so any transfer function of interest can be directly evaluated from these results. Three specific input–output pairs are considered in the following parts (a)–(c).

(a) *Flexible-base.* For the flexible-base case, the input is the free-field ground motion (u_g) and the output is the total motion at the roof level ($u_g + u_f + h\theta + u$). The transfer function is defined as the ratio of these two motions in the Laplace domain,

$$H_a(s) = 1 + \frac{\hat{u}_f + h\hat{\theta} + \hat{u}}{\hat{u}_g} = \frac{B_u B B_\theta}{C_s} \tag{12}$$

The poles for the flexible-base case are values of s for which $C_s = 0$. The algebraic solution of these roots[7] has the same form as equation (4), but with system frequency and damping ($\tilde{\omega}$ and $\tilde{\zeta}$) defined as

$$\tilde{\omega}^2 = \frac{\omega_\theta^2 \omega_u^2 \omega^2}{\omega^2 \omega_u^2 + \omega_\theta^2 \omega_u^2 + \omega^2 \omega_\theta^2} = \frac{1}{1/\omega^2 + 1/\omega_\theta^2 + 1/\omega_u^2} \tag{13a}$$

and

$$\tilde{\zeta} = \left(\frac{\tilde{\omega}}{\omega_u}\right)^3 \zeta_u + \left(\frac{\tilde{\omega}}{\omega}\right)^3 \zeta + \left(\frac{\tilde{\omega}}{\omega_\theta}\right)^3 \zeta_\theta \tag{13b}$$

These flexible-base parameters derived from the Laplace domain transfer function match those obtained from the frequency-domain transmissibility function by Luco.[8]

(b) *Pseudo flexible base.* The pseudo flexible-base case applies for a condition of partial base flexibility, representing base rocking only. This condition is important because the actual flexible-base parameters are often well-approximated by pseudo-flexible-base parameters. Furthermore, pseudo-flexible-base parameters can be used in procedures to estimate either fixed- or flexible-base parameters, as will be shown in the following section. For the pseudo-flexible-base case, the input is the total base translation ($u_g + u_f$) and the output is the total translation at the roof level ($u_g + u_f + h\theta + u$). Proceeding as in Part (a), the transfer function is

$$H_b(s) = \frac{\hat{u}_g + \hat{u}_f + h\hat{\theta} + \hat{u}}{\hat{u}_g + \hat{u}_f} = \frac{B_u B B_\theta}{C_s - s^2 B_\theta B} \tag{14}$$

The poles for the pseudo-flexible-base case are values of s for which the denominator is zero. The algebraic solution of these roots[7] yields the following system frequency and damping:

$$(\tilde{\omega}^2)^* = \frac{\omega_\theta^2 \omega_u^2 \omega^2}{\omega^2 \omega_u^2 + \omega_\theta^2 \omega_u^2} = \frac{1}{1/\omega^2 + 1/\omega_\theta^2} \tag{15a}$$

$$(\tilde{\zeta})^* = \left(\frac{\tilde{\omega}^*}{\omega}\right)^3 \zeta + \left(\frac{\tilde{\omega}^*}{\omega_\theta}\right)^3 \zeta_\theta \tag{15b}$$

From equation (15), the pseudo-flexible-base parameters are seen to be dependent on the rocking impedance of the foundation and the structural properties. It may be noted that flexible- and pseudo-flexible-base parameters are numerically similar when base rocking dominates the SSI (which, excepting broad, short structures, is often true).

(c) *Fixed base.* For the fixed-base case, the input is the sum of the total base translation and the contribution of base rocking to roof translation ($u_g + u_f + h\theta$), and the output is the total translation at the

Table I. Input and output pairs to evaluate system parameters for various conditions of base fixity

	System	Input	Output
a	Flexible base	u_g	$u_g + u_f + h\theta + u$
b	Pseudo flexible-base	$u_g + u_f$	$u_g + u_f + h\theta + u$
c	Fixed base	$u_g + u_f + h\theta$	$u_g + u_f + h\theta + u$

roof level ($u_g + u_f + h\theta + u$). Proceeding as with parts (a) and (b), the transfer function is

$$H_c(s) = \frac{\hat{u}_g + \hat{u}_f + h\hat{\theta} + \hat{u}}{\hat{u}_g + \hat{u}_f + h\hat{\theta}} = \frac{B_u B B_\theta}{C_s - s^2(B_\theta B + B B_u)} \tag{16}$$

The denominator in this case can be reduced to the product of a first- and second-order polynomial in s,

$$C_s - s^2(B_\theta B + BB_u) = [2s(\zeta_u \omega_u \omega_\theta^2 + \zeta_\theta \omega_\theta \omega_u^2) + \omega_\theta^2 \omega_u^2](s^2 + 2\zeta\omega s + \omega^2) \tag{17}$$

The second-order polynomial in equation (17) has the same form as the denominator of equation (3b). Hence, by analogy to that solution, the fixed-base parameters identified from this analysis are simply ω and ζ.

(d) *Summary*. The results presented in parts (a)–(c) show that a component of system flexibility is absent from the parametric system identification results when its associated motion is added to u_g in the input. For example, in Part (b), when the base translation motion is added to u_g in the input, the results represent only the structural flexibility and rocking foundation flexibility (i.e. base translation effects are 'removed'). Similarly, when base rocking and translation are added to u_g for the input in Part (c), the only remaining system flexibility is that of the structure.

The input and output required to evaluate system parameters for various conditions of base fixity are summarized in Table I. These results match those of Luco,[8] who solved the equations of motion (i.e. equations (6)) in the frequency domain to determine the input–output pairs necessary for evaluations of flexible-, pseudo-flexible- and fixed-base modal parameters using non-parametric identification procedures. The advantage of the present approach is that the more accurate estimates of transfer functions from parametric identification can be used to derive fundamental mode vibration parameters for various conditions of base fixity.

3. ESTIMATION OF FIXED AND FLEXIBLE-BASE FIRST-MODE PARAMETERS

Based on the results in Table I, it is necessary to have recordings of free-field, foundation and roof translations as well as base rocking to evaluate directly both fixed- and flexible-base modal parameters of a structure. If no recordings of roof translation are available, no modal parameters can be identified. However, if one of the other three motions is missing, the set of modal parameters not directly evaluated can be estimated. The two specific cases that will be considered here are missing base rocking motions (in which case fixed-base parameters are estimated), and missing free-field motions (in which case flexible-base parameters are estimated). The derivations in this section are made with respect to the single-degree-of-freedom model in Figure 2. For multi-mode structures, the quantities m and h are the effective mass and height, respectively. Although m and h are functions of the participation factors (which are not identified), the fundamental frequency and damping ratio are generally insensitive to reasonable estimates of m and h.

3.1. Estimation of fixed-base modal parameters (missing base rocking motions)

It will be shown in this section that fixed-base modal parameters for a structure can be estimated from 'known' flexible- and pseudo-flexible-base parameters. Hence, it is assumed that $\tilde{\omega}$, $\tilde{\zeta}$, $\tilde{\omega}^*$, and $\tilde{\zeta}^*$ have been determined from system identification procedures [Cases (a) and (b) in Table I]. Considering frequency first, equations (13a) and (15a) are re-written as

$$\frac{1}{\tilde{\omega}^2} = \frac{1}{\omega^2} + \frac{1}{\omega_\theta^2} + \frac{1}{\omega_u^2}$$
$$\frac{1}{(\omega^2)^*} = \frac{1}{\omega^2} + \frac{1}{\omega_\theta^2} \tag{18}$$

from which ω_u can be readily determined as

$$\frac{1}{\omega_u^2} = \frac{1}{\tilde{\omega}^2} - \frac{1}{(\tilde{\omega}^2)^*} \tag{19}$$

The ratio of ω_θ to ω_u is then taken using equation (10) to evaluate ω_θ as follows:

$$\left(\frac{\omega_\theta}{\omega_u}\right)^2 = \frac{k_\theta/mh^2}{k_u/m} = \frac{K_\theta \cdot \alpha_\theta(s) \cdot 1/h^2}{K_u \cdot \alpha_u(s)} \tag{20}$$

The quantities α_u and α_θ are dimensionless factors expressing the variation of foundation stiffness with the Laplace domain variable s,

$$\alpha_u = \frac{k_u}{K_u}, \quad \alpha_\theta = \frac{k_\theta}{K_\theta} \tag{21a}$$

For later reference, dimensionless foundation damping quantities β_θ and β_u can be similarly defined as,

$$\beta_u = \frac{c_u}{K_u r_1/V_S}, \qquad \beta_\theta = \frac{c_\theta}{K_\theta r_2/V_S} \tag{21b}$$

The quantities K_u and K_θ represent the static stiffness of the foundation in translation and rocking, respectively. The terms r_1 and r_2 are radii of equivalent circular foundations which match the area (A_f) and moment of inertia (I_f) of a noncircular foundation, i.e. $r_1 = \sqrt{A_f/\pi}$ and $r_2 = \sqrt[4]{4 \cdot I_f/\pi}$.

For surface foundations, equation (20) can be simplified using the impedance for a rigid circular disk foundation on the surface of an homogeneous, isotropic half-space. In this case, the static stiffnesses are $K_u = 8Gr_1/(2-\upsilon)$ and $K_\theta = 8Gr_2^3/3(1-\upsilon)$. Making these substitutions for K_u and K_θ in equation (20), the ratio of ω_θ to ω_u can be expressed as

$$\left(\frac{\omega_\theta}{\omega_u}\right)^2 = \left(\frac{r_2^3}{r_1 h^2}\right)\left(\frac{2-\upsilon}{3(1-\upsilon)}\right)\left(\frac{\alpha_\theta(s)}{\alpha_u(s)}\right) \tag{22}$$

Assuming the effective structure height, foundation geometry, and soil Poisson's ratio are known, the only unknown quantity in equation (22) is the ratio of dynamic factors $\alpha_\theta(s)/\alpha_u(s)$. In the frequency domain, the factors α_u and α_θ are commonly evaluated as functions of both frequency and soil hysteretic damping. Hence, such solutions may also be considered valid in the Laplace domain. As an approximation, the ratio $\alpha_\theta(s)/\alpha_u(s)$ can be computed for surface foundations using the frequency-domain dynamic impedance factors for a circular foundation on a uniform viscoelastic half-space derived by Veletsos and Verbic.[15] As shown in Figure 3, both factors are nearly unity for the low frequencies of most structures ($a_0 < 1$), so the α_θ/α_u ratio should not significantly affect the results.

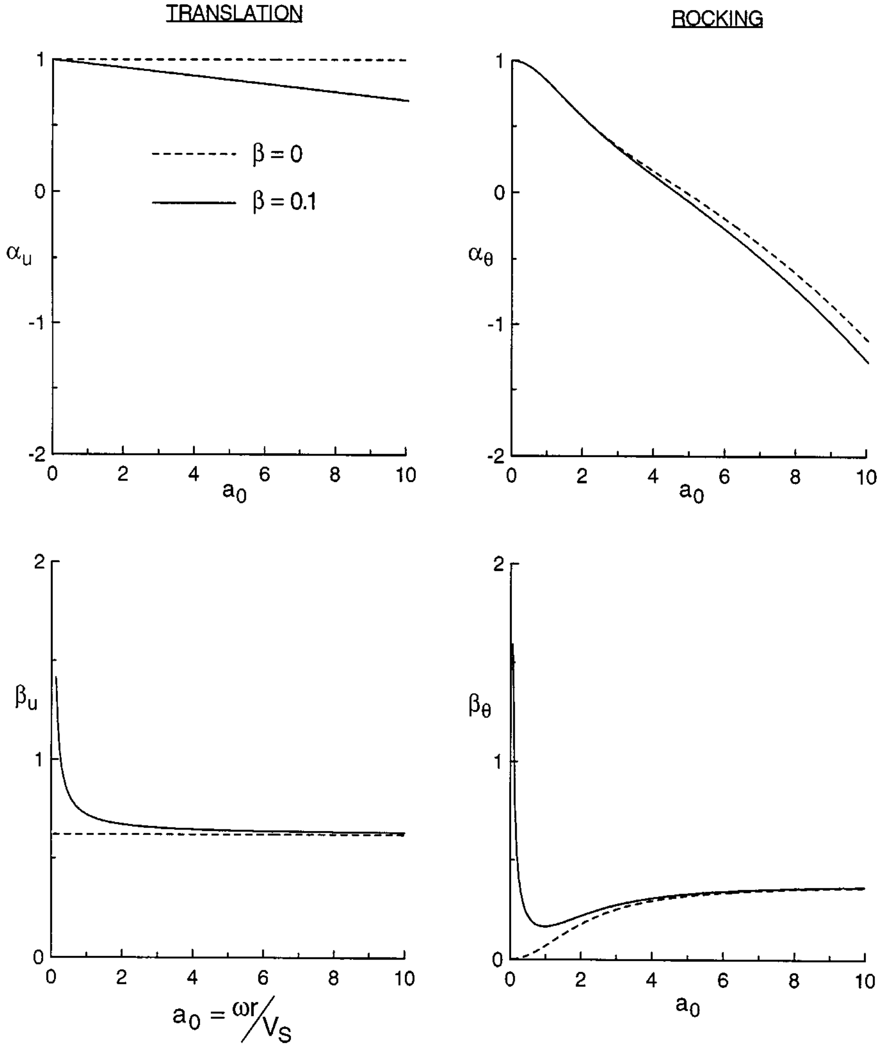

Figure 3. Foundation stiffness and damping factors for elastic and viscoelastic halfspaces, $\upsilon = 0.4$ (after Veletsos and Verbic, 1973)

For embedded foundations, equation (20) can be simplified using the impedance for a rigid circular disk foundation embedded into a homogeneous, isotropic half-space. Elsabee and Morray[16] found that the impedance of shallowly embedded ($e/r < 1$) foundations can be approximated by the product of the static stiffness for embedded foundations and α_u and α_θ factors for surface foundations. Static stiffness terms for shallowly embedded foundations ($e/r < 1$) can be approximately represented as follows:[17]

$$K_{U-E} = K_U\left(1 + \frac{2e}{3r}\right) \quad \text{and} \quad K_{\theta-E} = K_\theta\left(1 + 2\frac{e}{r}\right) \tag{23}$$

Coupling impedance terms are small relative to $(K_U)_E$ and $(K_\theta)_E$ for small embedment ratios (i.e. $e/r < 0{\cdot}5$). Using this formulation, the solution of equation (20) is reduced to

$$\left(\frac{\omega_\theta}{\omega_u}\right)^2 = \left(\frac{r_2^3}{r_1 h^2}\right)\left(\frac{2-\upsilon}{3(1-\upsilon)}\right)\left(\frac{\alpha_\theta(s)}{\alpha_u(s)}\right)\left(\frac{1+2(e/r)}{1+0{\cdot}67(e/r)}\right) \tag{24}$$

Obviously, use of the approximate solution in Reference 16 for embedded foundation impedance increases the error in estimating the ratio ω_θ/ω_u. However, for the moderate embedment ratios of many building structures ($e/r < 0{\cdot}5$), this error is expected to be small.

Once ω_θ is computed from equation (22) or equation (24), the fixed-base frequency can be readily computed from equation (15a) as

$$\frac{1}{\omega^2} = \frac{1}{(\tilde{\omega}^2)^*} - \frac{1}{\omega_\theta^2} \tag{25}$$

The evaluation of fixed-base damping involves more algebra, but the same principles apply. Equations (13b) and (15b) are used in conjunction with the ratio of the damping definitions for ζ_u and ζ_θ (equation (10)) to evaluate the unknown damping quantities ζ_u, ζ_θ, and ζ. The result is as follows:

$$\zeta = \frac{1}{C_1}\cdot\tilde{\zeta} - \frac{C_2}{C_1}\tilde{\zeta}^* \tag{26a}$$

where

$$C_1 = \left(\frac{\tilde{\omega}}{\omega}\right)^3 - C_3\frac{(\tilde{\omega}^*/\omega)^3}{(\tilde{\omega}^*/\omega_\theta)^3} \quad C_2 = \frac{C_3}{(\tilde{\omega}^*/\omega_\theta)^3}$$
$$C_3 = C_4\left(\frac{\tilde{\omega}}{\omega_u}\right)^3 + \left(\frac{\tilde{\omega}}{\omega_\theta}\right)^3 \quad C_4 = \frac{\omega_\theta}{\omega_u}\cdot\frac{\beta_u}{\beta_\theta}\cdot\frac{r_1^2 h^2}{r_2^4}\cdot\frac{3(1-\upsilon)}{2-\upsilon} \tag{26b–e}$$

The β_u and β_θ terms in C_4 can be approximated by the soil damping factors for a half-space in Veletsos and Verbic,[15] with appropriate corrections for foundation embedment, shape, and non-rigidity effects.[7] Even for surface foundations and low frequencies, these factors can be quite sensitive to frequency and hysteretic soil damping (e.g. Figure 3), so the evaluation of β_u/β_θ may be subject to significant errors. These errors are compounded for embedded foundations because radiation damping from basement-wall/soil interaction is difficult to quantify accurately. Hence, the estimates of fixed-base damping are subject to greater uncertainty than estimates of fixed-base frequency.

3.2. Estimation of flexible-base modal parameters (missing free-field motions)

For the derivation of flexible-base modal parameters, it is assumed that fixed- and pseudo-flexible-base system identification analyses have been performed [Cases (b) and (c) in Table I]. Hence, parameters $\tilde{\omega}^*$, $\tilde{\zeta}^*$, ω, and ζ are assumed known. The derivation follows the same steps as in Part (a). Using equation (15a), ω_θ is evaluated as

$$\frac{1}{\omega_\theta^2} = \frac{1}{(\tilde{\omega}^2)^*} - \frac{1}{\omega^2} \tag{27}$$

Frequency ω_u is computed from the ratio ω_θ/ω_u in equation (22) or equation (24), and the flexible-base frequency is determined directly from equation (13a).

For the case of damping, the algebra is less lengthy than Part (a). The first step is the calculation of ζ_θ from equation (15b),

$$\zeta_\theta = \frac{\tilde{\zeta}^* - (\tilde{\omega}^*/\omega)^3\,\zeta}{(\tilde{\omega}^*/\omega_\theta)^3} \tag{28}$$

Damping ζ_u is then evaluated from the following,

$$\zeta_u = \zeta_\theta \frac{\omega_\theta}{\omega_u} \cdot \frac{\beta_u}{\beta_\theta} \cdot \frac{r_1^2 h^2}{r_2^4} \cdot \frac{3(1-\upsilon)}{2-\upsilon} \tag{29}$$

and the flexible-base damping is determined directly from equation (13b).

The same limitations on solution accuracy that were stated in part (a) apply to the estimation of flexible-base parameters. Namely, the accuracy of estimated flexible-base frequency is generally better than the accuracy of flexible-base damping, especially for embedded foundations.

4. VERIFICATION OF PARAMETER ESTIMATION PROCEDURES

4.1. Data set and errors in the analyses

A study by the first author[18] evaluated soil–structure interaction effects from strong motion recordings at 58 building sites. Of these, 11 sites (with a combined total of 19 data sets) had complete instrumentation configurations which enabled a direct identification of fixed- and flexible-base parameters. The system identification results from these sites are used to confirm the accuracy and limitations of the parameter estimation procedures by comparing the identified and estimated first-mode parameters. Table II summarizes the verification of the estimation procedures for the eleven sites.

The conditions necessary to apply the parameter estimation procedures are that $\tilde{f} < \tilde{f}^*$ for estimating fixed-base parameters and $\tilde{f}^* < f$ for estimating flexible-base parameters, where $f, \tilde{f}^*$, and $\tilde{f}$ are first-mode fixed, pseudo-flexible, and flexible-base frequencies, respectively. For the 11 sites with complete instrumentation sets, these conditions were met for 11 of the 19 data sets for estimating fixed-base parameters, and in 17 of 19 data sets for estimating flexible-base parameters.

Each site which failed the $\tilde{f} < \tilde{f}^*$ or $\tilde{f}^* < f$ criteria had negligible period lengthening, and the criteria were not met as a result of small numerical errors inherent to all system identification results. One source of error is inadequate model structure, which principally consists of non-optimization of the model order (J) in the parametric system identification. To minimize such errors, J was selected to minimize deviations between the model output and roof recording, while not over-parameterizing the model in such a way as to cause pole-zero cancellation in the transfer function. In addition, a second constraint was imposed that required the input-output pairs for a given direction in a building have the same order, J. This was enforced so that variations between modal parameters for different conditions of base fixity are reflective of true SSI effects and are not by-products of the analyses. As a result of this constraint on J, models for some input–output pairs may not be optimal with respect to the minimization of model error.

A second source of error is random disturbances in the output data. These are quantified as part of the identification procedure, and are indicated as standard deviations of the estimate along with the mean values in Table II. The coefficients of variation associated with these errors are often about 0·5 to 1·5 per cent for frequency and 5 to 15 per cent for damping, but are higher in some cases. Random disturbance errors for the identified parameters in Table II are typically lowest for pseudo-flexible-base parameters and highest for either the flexible- or fixed-base cases. The additional uncertainty for the flexible-base case is likely associated with spatial incoherence effects, which may 'contaminate' free-field recordings relative to the 'true' free-field motion for the site. Stewart[7] attempted to minimize this uncertainty by developing criteria for site selection which maximize the coherence of free-field motions at the structure and free-field locations, while also minimizing the effects of structural vibrations on free-field recordings. For the fixed-base case, an additional variability arises from the subtraction of one vertical foundation level record from another in order to obtain base rocking. This subtraction can, in some cases, result in small signals and reduced signal-to-noise ratios. However, as illustrated below, for structures subject to significant base rocking, the difference between the vertical signals is substantial, and random disturbance errors in vibration parameters identified using these signals are small.

Table II. Comparison of system identification results (mean and standard deviation) and estimated fixed- and flexible-base fundamental-mode parameters

				Pseudo-flex.-base		Fixed-base (Sys. ID)		Fixed-base (estimated)		Flex.-base (Sys. ID)		Flex.-base (estimated)	
Site	e/r_1	EQ[1]	Dir	$\tilde{f}^*$ (Hz)	$\tilde{\zeta}^*$ (%)	f (Hz)	ζ (%)	f (Hz)	ζ (%)	$\tilde{f}$ (Hz)	$\tilde{\zeta}$ (%)	$\tilde{f}$ (Hz)	$\tilde{\zeta}$ (%)
A4	0	LP	TR	0.41 ± 0.01	7.5 ± 2.2	0.41 ± 0.02	7.4 ± 7.3	0.44	0	0.40 ± 0.02	13.0 ± 4.4	0.41	7.6
			L	0.37 ± 0.01	7.1 ± 2.0	0.38 ± 0.01	5.9 ± 1.7	—	—	0.39 ± 0.02	12.8 ± 4.9	0.37	8.1
A8	0	LP	TR	6.07 ± 0.01	2.9 ± 0.1	6.32 ± 0.02	2.2 ± 0.1	6.42	3.2	5.44 ± 0.02	4.8 ± 0.1	5.60	15.2
A23	0.65	NR	TR	1.11 ± 0.01	6.6 ± 0.4	1.21 ± 0.01	6.9 ± 0.5	—	—	1.12 ± 0.01	5.5 ± 0.5	1.10	7.6
A24	0	NR	L	2.50 ± 0.1	5.4 ± 0.1	3.63 ± 0.02	6.5 ± 0.2	3.19	5.4	2.27 ± 0.01	6.1 ± 0.3	2.21	5.6
A25	0	NR	TR	0.79 ± 0.02	31.1 ± 6.4	0.79 ± 0.02	29.3 ± 6.0	—	—	0.84 ± 0.03	27.1 ± 6.6	0.79	72
A31	0	NR	TR	1.18 ± 0.00	4.9 ± 0.2	1.34 ± 0.01	8.5 ± 0.5	1.27	7.1	1.16 ± 0.01	3.2 ± 0.2	1.15	3.0
A33	0.11	WT	L	0.82 ± 0.01	2.8 ± 0.3	0.82 ± 0.01	3.0 ± 0.3	—	—	0.83 ± 0.02	3.5 ± 0.7	—	—
		NR	L	0.82 ± 0.01	3.1 ± 0.5	0.82 ± 0.01	3.4 ± 0.5	—	—	0.82 ± 0.02	4.6 ± 0.6	—	—
A37	0.12	RD	TR	1.68 ± 0.01	4.2 ± 0.3	1.71 ± 0.01	3.7 ± 0.3	1.70	4.3	1.66 ± 0.02	4.3 ± 0.5	1.64	10.9
		WT	TR	1.57 ± 0.01	4.9 ± 0.2	1.58 ± 0.01	5.0 ± 0.2	1.58	5.0	1.55 ± 0.02	4.4 ± 0.4	1.54	5.4
		UP	TR	1.28 ± 0.01	7.3 ± 0.4	1.29 ± 0.01	7.8 ± 0.4	—	—	1.31 ± 0.02	4.7 ± 0.6	1.26	4.7
		LD	TR	1.16 ± 0.01	12.7 ± 1.3	1.17 ± 0.02	12.5 ± 1.3	1.17	12.9	1.16 ± 0.02	11.2 ± 1.4	1.15	17.8
		NR	TR	1.32 ± 0.01	6.4 ± 0.2	1.34 ± 0.01	6.9 ± 0.2	1.32	6.5	1.32 ± 0.01	4.6 ± 0.4	1.29	2.9
A43	0.16	LD	TR	0.78 ± 0.01	12.6 ± 0.9	0.80 ± 0.01	13.2 ± 1.0	—	—	0.78 ± 0.01	11.5 ± 1.0	0.76	13.3
		NR	TR	0.82 ± 0.01	7.1 ± 0.5	0.85 ± 0.01	7.1 ± 0.5	—	—	0.83 ± 0.01	6.6 ± 0.5	0.80	12.3
A45	0	NR	TR	1.49 ± 0.02	3.7 ± 0.5	1.88 ± 0.02	5.7 ± 0.7	1.86	6.6	1.41 ± 0.02	4.0 ± 0.4	1.41	5.2
A46	0.92	LT7	TR	2.37 ± 0.02	16.7 ± 1.1	8.5	3	*	*	2.04 ± 0.03	30.6 ± 2.8	2.19	30.2
			L	2.42 ± 0.02	12.9 ± 0.8	9.0	3	*	*	2.25 ± 0.03	31.0 ± 3.0	2.24	30.0

— no solution possible as $\tilde{f} > \tilde{f}^*$ for fixed-base, or $\tilde{f}^* > f$ for flexible-base

* No solution

[1] Earthquakes: 1985 Redlands (RD), 1987 Whittier (WT), 1989 Loma Prieta (LP), 1990 Upland (UP), 1992 Landers (LD), 1994 Northridge (NR), Lotung event LSST07 (LT7)

4.2. Example of system identification

Site A45 is a 12-storey hotel with no basement that resists lateral loads by reinforced concrete shear walls. The building layout and instrumentation are shown in Figure 4. The foundation consists of 19–20 m long reinforced concrete tapered piles, the heads of which are cast into 0·8 m thick caps interconnected by tie beams. Selected ground motions recorded at the site in the building's transverse (EW) direction during the 1994 Northridge earthquake are shown in Figure 5(a)–5(c). Figure 5(d) shows the contribution of base rocking to roof displacements by comparing relative lateral roof displacements with the product of base rotation and building height. The calculated rocking displacements are in phase with the relative roof displacements and have about 30 to 40 of the roof displacement amplitude, indicating that base rocking significantly influenced the roof recordings.

Figure 6 presents non-parametric transmissibility function amplitudes for the flexible-, pseudo-flexible, and fixed-base cases. First-mode parameters identified by parametric system identification analyses are presented in Table II for site A45. Figure 7 presents the roof components of flexible- and fixed-base transfer function amplitudes plotted against the scaled Laplace-domain quantities $b = \sqrt{s \cdot s^*}$ and $a = -\mathrm{Re}(s)/b$. It may be noted from equation (5) that at a particular pole j, $b = \omega_j$ and $a = \zeta_j$ (i.e. modal frequency and damping for mode j). SSI exerts a moderate effect on the structural response at site A45, as evidenced by the

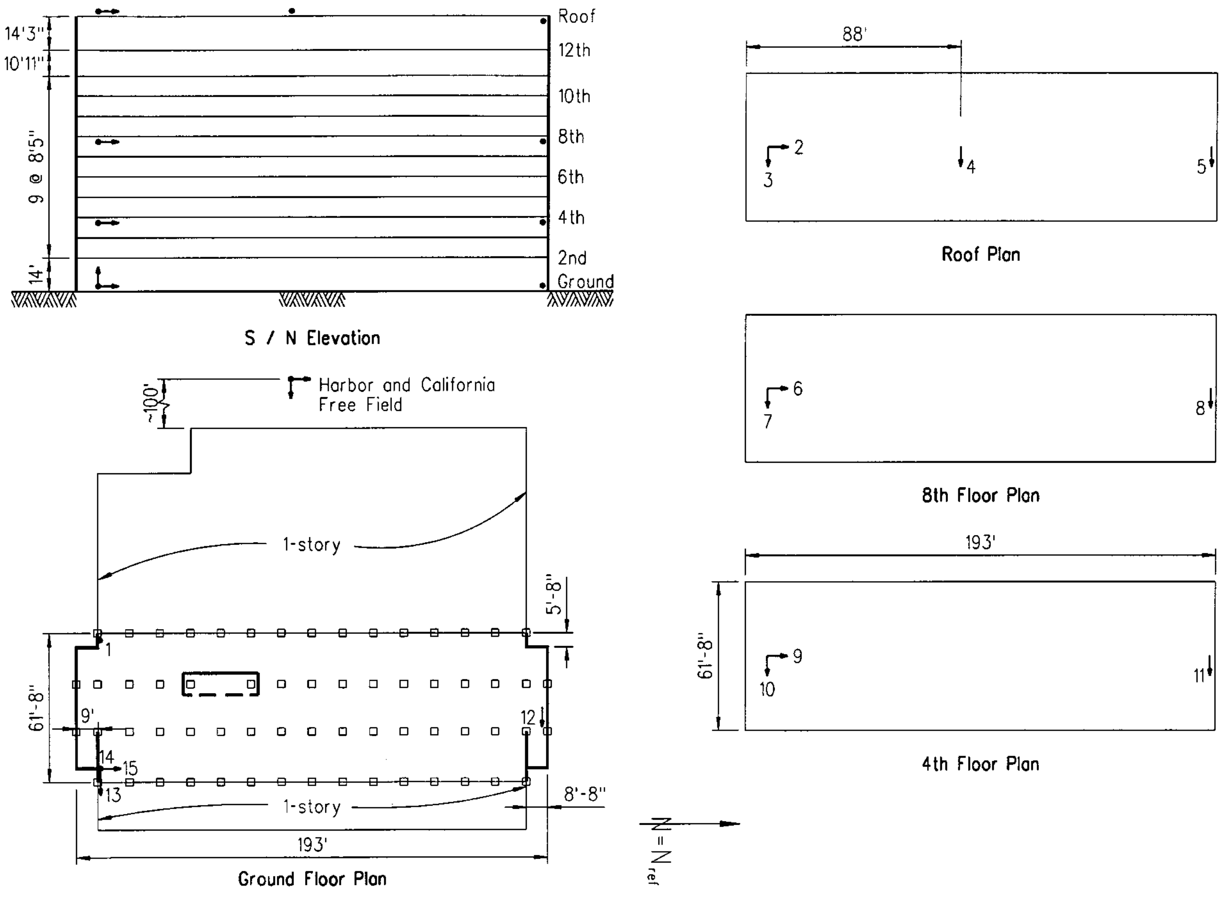

Figure 4. Structural configuration and instrument layout at site A 45

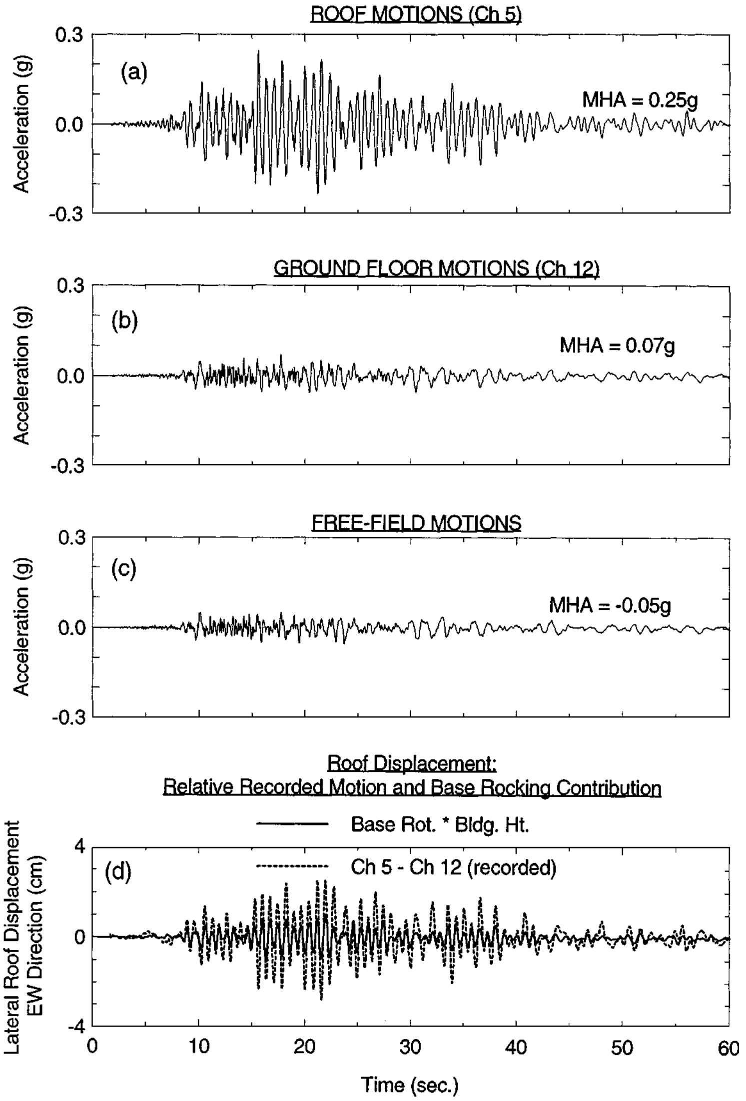

Figure 5. (a)–(c) Recording of roof, ground floor, and free-field translations at site A45 during the 1994 Northridge earthquake. (d) Comparison of relative roof/base translations and base rocking contribution to roof displacements

shift in first-mode frequency shown in Figures 6 and 7. The SSI effect is primarily associated with base rocking, as evidenced by the contrast between fixed- and pseudo-flexible-base parameters (e.g. $f = 1{\cdot}88$ Hz and $\tilde{f}^* = 1{\cdot}49$ Hz) and the relative displacements in Figure 5(d).

If the base rocking motions are neglected, invoking the parameter estimation procedures described in Section 3 yields reasonably accurate values of fixed-base frequency and damping (i.e. system identification results are $f = 1{\cdot}88$ Hz and $\zeta = 5{\cdot}7$ per cent, estimates are $f = 1{\cdot}86$ Hz and $\zeta = 6{\cdot}6$ per cent). A similar conclusion is reached for the flexible-base case if free-field motions are neglected (i.e. system identification results are $\tilde{f} = 1{\cdot}41$ Hz and $\tilde{\zeta} = 4{\cdot}0$ per cent, estimates are $\tilde{f} = 1{\cdot}41$ Hz and $\tilde{\zeta} = 5{\cdot}2$ per cent).

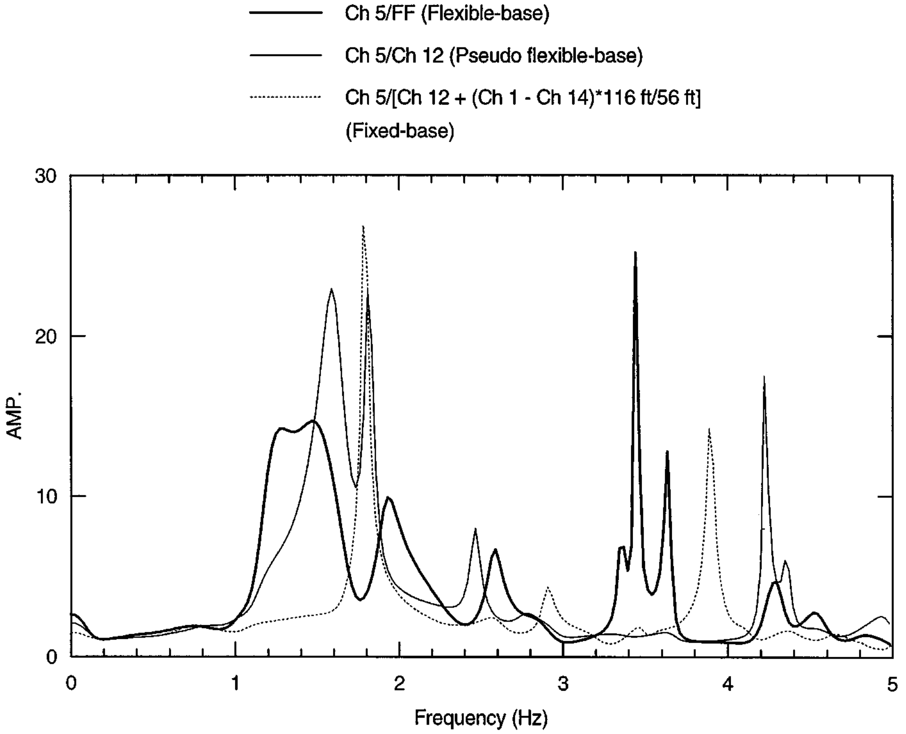

Figure 6. Transmissibility functions for transverse response of site A45 during the 1994 Northridge earthquake

4.3. Overview of results for parameter estimation procedures

Table II presents the full data set of sites with instrumentation of the building, foundation, and free-field. Frequencies estimated using the procedures described in Section 3 are reasonably accurate relative to identified values both at sites where inertial interaction effects are significant (e.g. A24, A46) as well as sites where these effects are minor (e.g. A8, A37). The estimates of fixed-base damping (ζ) differed from identified values by absolute differences of about 1 to 2 per cent (except at site A4, where random disturbance errors in ζ are large). At several sites with significant interaction, the estimated ζ values are closer to "known" ζ values than are the pseudo-flexible-base damping ($\tilde{\zeta}^*$) values (e.g. A31, A45). By contrast, the estimation of flexible-base damping ($\tilde{\zeta}$) is highly sensitive to differences between ζ and $\tilde{\zeta}^*$, and where such differences are small (i.e. within the range of uncertainty of the system identification results), $\tilde{\zeta}$ values can be significantly overestimated [e.g. A8, A25, A37 (rd), A37 (up), A43 (nr)]. However, when the difference between ζ and $\tilde{\zeta}^*$ values is large (e.g. A24, A31, A45, A46), the flexible-base damping is generally well predicted. Overestimated $\tilde{\zeta}$ values are usually fairly obvious from comparisons with $\tilde{\zeta}^*$ values. For such cases, the results in Table II indicate that flexible-base damping is generally better estimated by $\tilde{\zeta}^*$ values.

The fixed-base parameters could not be estimated for site A46. A large increase in f relative to $\tilde{f}$ is predicted by the procedure (which is consistent with the unusually large SSI effect at this site), but the estimate is undefined because $\tilde{\omega}_\theta < \tilde{\omega}^*$ (i.e. see equation (25)). In this case, the estimation of f is highly sensitive to the relatively small difference between $\tilde{f}$ and $\tilde{f}^*$, and the instability of the estimate is likely associated with small numerical errors in the identified $\tilde{f}$ and $\tilde{f}^*$. Errors associated with model structure are unusually large at this site due to the stiff structural response (i.e. the system identification results in this case were sensitive to the order of the model, J). In contrast, the flexible-base parameters were accurately estimated because the estimate is based on the relatively large differences between the fixed and pseudo flexible-base parameters.

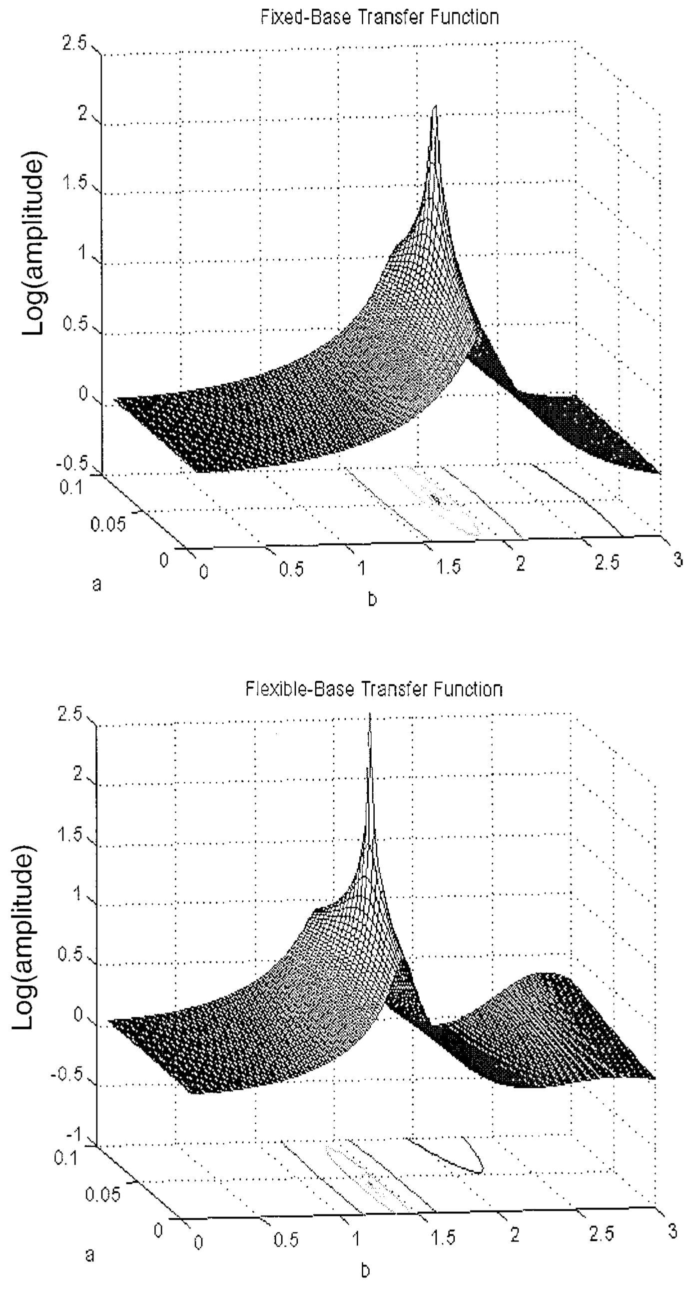

Figure 7. Fixed- and flexible-base transfer function surfaces for Site A45

5. CONCLUSIONS

Parametric system identification is a useful tool for evaluating the effects of soil–structure interaction (SSI) on the seismic response of structures. For a complete identification of both fixed- and flexible-base modal parameters, strong motion recordings of the base rocking as well as lateral roof, foundation, and free-field motions are required. The input–output pairs used in a parametric procedure formulated in the Laplace

domain to identify fixed- and flexible-base modal parameters were shown to be the same as those derived by Luco[8] for non-parametric procedures performed in the frequency domain. Procedures were also developed for estimating either fixed- or flexible-base parameters for sites lacking certain strong motion recordings, extending substantially the number of sites for which SSI effects can be empirically quantified.

These estimation procedures generally provide reasonable estimates of system identification results at eleven sites where complete instrumentation sets are available. Comparisons of estimated and "known" first-mode parameters show that (1) fixed- and flexible-base frequencies are reliably predicted by the parameter estimation procedures, (2) estimated fixed-base damping ratios are fairly accurate (absolute difference between estimated and actual values of about 1 to 2 per cent), and (3) flexible-base damping is generally well predicted when SSI effects are significant, but it can be overpredicted when SSI effects are modest. For cases with modest SSI effects but large differences between estimates of flexible-base damping and fixed-base damping, the flexible-base damping is better estimated by the pseudo-flexible-base damping.

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